Electrostatics

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Magnetostatics

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The "Mass" of the Photon - Magnetic Fields in Superconductors

If **photons** had a nonvanishing **mass**, the electromagnetic fields would show different characteristics than thc to: a wavelength dependence of the **speed of light**, modifications of Coulomb's and Ampères law and thus di charges and dipoles and so on. Find out more about the photon mass and how related theories can be u **superconductivity**.

Mathematical Description: The Proca Equations

In this first part we will loosely follow some lines of the very interesting review "The mass of the photon" by Tu et al., Repc in Physics **68** (2005) (PDF). More information about superconductivity can be found for example in Jackson's "Classical E or Kittel's "Introduction to Solid State Physics" from an electrodynamics or solid states background, respectively. The Maxwells equations allowing massive photons was introduced by the romanian physicist **Alexandru Proca** in the early t and had a deep impact on **particle physics**. We will see in the second part that the mathematical framework Proca dev be used as a phenomenological description of **superconductivity** developped by the **London brothers**. The so-called **P** incorporate a photon mass m_γ into Maxwell's equations: Both Gauss's and Ampères law are modified (statics!):

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\epsilon_0} - \mu_\gamma^2 \phi(\mathbf{r})$$
$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{j}(\mathbf{r}) - \mu_\gamma^2 \mathbf{A}(\mathbf{r}) \ .$$

For visibility, we may use the **abbreviation** $\mu_\gamma = m_\gamma c/\hbar$. Note that the latter equations can be derived from the Lagra the fields with an extra term $\propto \mu_r A_\mu(x^\nu) A^\mu(x^\nu)$ in relativistic notation taking the static limit. In the Proca-equations $\phi(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ are written explicitly and obtain a **direct physical meaning** which is otherwise not the ca electrodynamics.

Let us concentratete on the **magnetic** part and how the fields are changing due to the photon mass. Using $\mathbf{B}(\mathbf{r}) = \nabla \times$ with $\nabla \times \nabla \times \mathbf{A}(\mathbf{r}) = \nabla(\nabla \cdot \mathbf{A}(\mathbf{r})) - \Delta \mathbf{A}(\mathbf{r})$ in Coulomb gauge, $\nabla \cdot \mathbf{A}(\mathbf{r}) = 0$,

$$\left(\Delta - \mu_\gamma^2\right) \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{j}(\mathbf{r}) \ .$$

This linear partial differential equation can be conveniently solved using the **Green's function** for which

$$\mathbf{A}(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}') \, \mathbf{j}(\mathbf{r}') \, dV' \ .$$

However, in contrast to the $\propto 1/|\mathbf{r} - \mathbf{r}'|$-dependency in usual electro- and magnetostatics, we obtain

$$G(\mathbf{r}, \mathbf{r}') = \frac{\mu_0}{4\pi} \frac{\exp(-\mu_\gamma |\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} \ .$$

This Green function with its exponential decay is characteristic for so-called **Yukawa potentials**. Now, for the magnetic di

$$\mathbf{m} = \frac{1}{2} \int \mathbf{r}' \times \mathbf{j}(\mathbf{r}') \, dV' \ ,$$

we find the **dipole field** for the magnetic induction to be

$$\mathbf{B}_\mathrm{D}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{e^{-\mu_\gamma r}}{r^3} \left\{ \left[1 + \mu_\gamma r + \frac{1}{3}\mu_\gamma^2 r^2\right] \left[3\left(\mathbf{m} \cdot \mathbf{e}_r\right) \mathbf{e}_r - \mathbf{m}\right] - \frac{2}{3}\mu_\gamma^2 r^2 \mathbf{m} \right\} \ .$$

This dipole field is thus modified with respect to the usual result without photon mass. The **first term** leads to a strengther dipole, $\mathbf{m} \to \left[1 + \mu_\gamma r + \frac{1}{3}\mu_\gamma^2 r^2\right] \mathbf{m}$but we can also see the characteristic **Yukawa fall-off** in the pre-factor.

The **second term**, however, leads to a further contribution to the magnetic induction. Assuming that the magnetic fielc basically a dipole, **Schrödinger** suggested that the ratio of the ordinary dipole moment to the second term in $\mathbf{B}_\mathrm{D}$ should with respect to the usual modified first term. In 1955, Bass and Schrödinger analysed magnetic survey data from 192 **upper limit** of $m_\gamma \lesssim 10^{-47}$ g. A lot of studies followed employing different methods like large-scale observations of m the universe. All investigastions resulted in a maximum limit of the photon mass more or less close to the official value of group, $m_\gamma \leq 10^{-49}$ g. However, the question if the photon has a mass is still open and may never be solved conclusive ever find the photon mass to be exactly zero? Maybe you have the right idea - Stockholm is calling!

Magnetic Fields in Superconductors: The London Penetration De

Now, how can we use massive photons to **phenomenologically** describe superconductivity? The **London theory of su** gives an explanation of the so-called **Meissner effect**. This effect, discovered in the thirties of the last century, states field can only have a finite penetration into a superconductor, which the Londons explained assuming that ph superconductor acquire an effective mass. Of course, this explanation links the **Proca** form of the magnetic field to the Let us understand the finite penetration depht of the magnetic fields into superconductors in the following.

We start with an expression for the current in a non-relativistic conductor given by

$$\mathbf{j}(\mathbf{r}) = \mathbf{j}_{\text{cond}}(\mathbf{r}) + \mathbf{j}_{\text{ext}}(\mathbf{r}) = q n_q \mathbf{v}(\mathbf{r}) + \mathbf{j}_{\text{ext}}(\mathbf{r}) \ .$$

For the moment, let us not consider any external charges but only the conductive contribution inside the superco $\mathbf{j}_{\text{ext}}(\mathbf{r}) = 0$. The **generalized momentum** of a charged particle in an electromagnetic field is given by $\mathbf{p} = m_q \mathbf{v} + \frac{q}{c}$ express the conductive current as

$$\mathbf{j}(\mathbf{r}) = \frac{q n_q}{m_q}\left(\mathbf{p}(\mathbf{r}) - \frac{q}{c}\mathbf{A}(\mathbf{r})\right) \ .$$

Now the Londons assumed that the superconducting state is characterized by a **vanishing generalized momentu** assumption got its theoretical foundation later on quantum mechanical grounds and implies here

$$\mathbf{j}(\mathbf{r}) = -\frac{q^2 n_q}{m_q c}\mathbf{A}(\mathbf{r}) \ .$$

Now inserting this current into Ampère's law, we obtain

$$\nabla \times \mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{j}(\mathbf{r}) = -\mu_0 \frac{q^2 n_q}{m_q c}\mathbf{A}(\mathbf{r}) \ ,$$

$$-\Delta \mathbf{A}(\mathbf{r}) = -\mu_0 \frac{q^2 n_q}{m_q c}\mathbf{A}(\mathbf{r})$$

using $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$ and $\nabla \cdot \mathbf{A}(\mathbf{r}) = 0$ With the **abbreviation**

$$\mu_{\text{L}}^2 = \mu_0 \frac{q^2 n_q}{m_q c}$$

we find, reincorporating an external current,

$$\left(\Delta - \mu_{\text{L}}^2\right)\mathbf{A}(\mathbf{r}) = -\mathbf{j}_{\text{ext}}(\mathbf{r}) \ .$$

So, in the end, the conductive current we introduced can be interpreted as an **effective photon mass** and leads t formulation as the one given by Proca. However, in the London theory, μ_γ is replaced by μ_{L}. Both μ's have the dimensi length and $\lambda_{\text{L}} = 1/\mu_{\text{L}}$ is termed the **London penetraton depth**. For low-temperature superconductors, μ_{L} is in the or Now we have everything at hand to understand why λ_{L} is indeed the penetration depth of the magnetic field - We str you to find out why solving "Superconductors and Their Magnetostatic Fields". Ok, you may also simply look up the soluti

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